1. INTRODUCTION

This work concerns the problem of extracting climate signals or climatologies from observations collected at fixed sites. Ground-based observing stations are typically distributed unevenly and sparsely, leaving many scales of spatial variability poorly sampled. Moreover, the stations typically do not operate over a common time period of adequate length—and most don’t collect data uniformly even when they are operating normally. This is especially serious from a climate-study perspective if, for example, weather conditions influence the decision whether or not to take an observation (see Elliot and Ross, 2000).

The most widely-used approach to obtain spatio-temporal signals from such data is to begin by mapping the observations \( Z(x) \) onto a regular grid using spatial analysis methods. Desired climate signals \( \alpha \) are then obtained from the series of maps. I will use the term “data mapping” to describe this procedure, arguing that clear benefits are from the series of maps. I will use the term “data mapping” services evaluated directly at the observing locations without mapping possible from an approach in which the data, an alternative I will call “signal mapping.” Here the benefits are discussed briefly, and an explicit method presented for implementing signal mapping that exhibits these benefits in several tests.

One disadvantage of data mapping is inaccuracy. Any linear method for predicting \( Z(x) \) from discrete observations \( z \) can be written (dropping arguments) in terms of a weight vector \( \beta(\alpha) \):

\[
\hat{Z} = \beta \cdot z.
\]

An “optimal” (e.g. best linear unbiased prediction, BLUP) data-mapping method typically chooses \( \beta \) satisfying

\[
\frac{\partial}{\partial \beta} (\hat{Z} - Z) = 0,
\]

where brackets denote ensemble mean or expectation value, and \( \beta \) indicates an optimal estimate (in the least-squares sense) of the quantity beneath. But if what you really want is a climate statistic—for simplicity, say, the time average of \( Z, \hat{Z} \) where

\[
Z \equiv \bar{Z} + Z',
\]

then the best estimate of \( Z \) is one obtained from \( \beta_1 \) for which

\[
\frac{\partial}{\partial \beta_1} (\hat{Z} - Z) = 0.
\]

Using (3), (2) can be rewritten as

\[
\frac{\partial}{\partial \beta} (\hat{Z} - Z)^2 + (\hat{Z}' - Z') = 0.
\]

Equation (5) just says that the “optimal” weights \( \beta \) try to minimize the errors in \( \hat{Z} \) and \( Z' \). However, since \( \hat{Z} \) can typically be estimated with much smaller error than is possible for \( Z' \), the second term dominates (5). Thus the weights \( \beta \) and resulting sample mean of the BLUP’s, \( \hat{Z} \), may differ substantially from the weights \( \beta_1 \) and the best linear estimate \( \hat{Z} \) that you actually want.

A second benefit of “signal mapping” is error estimation. It is nearly impossible to estimate accurately the error of a functional applied to a series of BLUP’s, even when error for each BLUP is available, since the latter are not independent. Thus one almost never sees error bars quoted for quantities estimated from gridded analyses, severely limiting the value of such estimates. On the contrary, the procedures discussed here allow straightforward estimation of uncertainty in \( \alpha \).

A third benefit is the ability to treat changes or relocations of instrumentation. This becomes clear below.

2. A MOTIVATING EXAMPLE: TIDES

I first demonstrate the possibilities of signal mapping using a simple example, the estimation of the migrating atmospheric tide. The tidal wind has previously been estimated from models and from data at individual stations, but here it is mapped globally from data.

The mapping procedure chosen here is statistical interpolation or Kriging, described for example in Daley (1981). Fig. 1 compares the estimation of this tide in the lower stratosphere, by a) data mapping (mapping the wind data first and then finding the tide in the mapped estimates), and b) signal mapping (finding the tide at each station by taking day-night differences where they are available, then mapping the tidal amplitude). An alternative signal-mapped result is also shown in which the tide is assumed to be a function only of the local time, so that each tide estimate can be used in two places on the globe. Finally a GCM result is shown.

The signal-mapped tides agree remarkably closely with the model result, but the data mapped tides are terrible. One reason for this is that the tidal oscillation is much more spatially coherent, and much better captured by a long dataset, than is the continuously evolving wind field; thus, the optimal map of the wind field at each synoptic time is too conservative and ends up unnecessarily discarding information vital to estimating the tide. Similar problems would occur regardless of the mapping or...
curve-fitting procedure (unless external information were used such as model forecasts). Other reasons for the poor performance will emerge below.

3. IUK METHOD

The above example shows the benefit—in principle—of signal mapping. Unfortunately, most signals are not so easy to separate from unwanted variability as are tides. Here a method that implements this approach for general signals is described briefly (more information is available from the author) and its performance discussed. The data must come from an array of fixed sites, and the signals sought must be additive, but any amount of data can be missing.

The method is an iterative form of universal kriging (e.g., Cressie, 1993), hence dubbed IUK. Universal kriging is the representation of the data as a parametric model plus a random process,

$$ Z = \mu + \epsilon $$

where each of these quantities is a function of space and time. The parametric model $\mu$ will represent the smooth variations, and $\epsilon$ the deviations from $\mu$. $\mu$ is a linear superposition of basis functions:

$$ \mu(s, t) = \sum_{i=1}^{m} \alpha_i f_i(s, t) + \sum_{j=1}^{n} b_j g_j(s, t). $$

The variables $x(x)$ and $t$ are the discrete space and time coordinates at which data are potentially available. The basis functions $f$ should be chosen by the analyst to represent the desired signal pattern; for example, if the trend is desired then one or more of the $f_i$ should be linear functions of time. The additional functions $g$ represent other large-scale variability, and in general may be determined empirically or a priori; here, they are set equal to the leading EOFs of the residual from a fit to $f$. $Z$, $f$, and $g$ can be scalars or vector quantities. The field $\epsilon$ is modeled as a Gaussian random field (GRF) having a heterogeneous and stationary autocovariance function $\sigma_\epsilon(dx, dt)$ as in standard statistical interpolation (e.g., Daley, 1991).

Note that (7) is a generalization of curve- or spline-fitting techniques (which would simply treat $\epsilon$ as a white noise process), and standard statistical interpolation (SI) techniques (which would treat $\mu$ as a constant). North et al. (1995) provided an analytical solution for the special case where $\epsilon$ is white, all $\{f, g\}$ are known a priori (typically from a numerical model), and the data are departures from a known climatology. Here these conditions are relaxed: $\sigma_\epsilon(dx, dt)$ is fitted to $\epsilon$, $g$ can be determined by the data, and the climatology is estimated.

The complete set of observables $Z$ at all $s, t$ is made up of an available subset $Z_A$ and a missing or unknown subset $Z_M$ (similar notation is used for $\epsilon$). A straightforward method is assumed to be available for estimating $a$ from a complete dataset, $Z$; the goal here is to find the best estimates of $a$ given only $Z_A$.

IUK proceeds by a series of steps illustrated in Fig. 2. This series of steps is a direct implementation of the EM algorithm due to Dempster et al. (1977). This algorithm converges absolutely to estimates of parameters $a$ that are maximum-likelihood given $f, g$, and the fitted parameterization of $\sigma_\epsilon(dx, dt)$. Note that at no time is any variable evaluated anywhere except at the station locations $s$.

With appropriate construction of $\mu$, however, parameters $a$ and/or functions $f, g$ can ultimately be mapped from $s$ onto $x$ (e.g., Fig. 1b,c).
5. UNCERTAINTY ESTIMATION

Determination of uncertainty in estimated $a$ is straightforward using bootstrap methods. If $a_i$ are identified with stations $i$ (as in test 2) and are mapped onto a regular grid using statistical interpolation, this technique also provides uncertainty maps (e.g., Daley, 1991).

6. APPLICATIONS

Though it can be used to retrieve any additive signal, IUK is particularly well-suited for obtaining accurate climatologies in the presence of coherent variations or trends. In contrast, nearly all popular signal detection methods depend on $a$ priori knowledge of the climatology before the search for oscillations, trends, etc. can even begin; if the climatology is not known well enough, their signal estimates may be adversely affected. In this case IUK can and should be used to retrieve the time-dependent signals at the same time it establishes the climatology. IUK is particularly useful when no suitable “reference period” exists over which all stations report, or where this period is of insufficient length. It can tolerate any missing-data pattern while squeezing the most out of all the data available.

IUK is also useful when stations move or change instrumentation, since the two locations or instrument types can be treated as separate values of $a$ even if they are at the same $x$. The effect of the instrument change on $a$, if any, is then automatically estimated by the method if appropriate basis functions are provided.

One application where the climatology is important is in cross-equatorial flow in the stratosphere, the sign of which is not yet known at some levels (results shown at the meeting). Another promising application is the calculation of wind divergence. This important quantity is strongly related to tropospheric dynamics, and is notoriously difficult to estimate. Evidence is shown in the talk that in some regions, IUK estimates of upper-tropospheric divergence may be superior to those obtained from operational analyses. Recent results will also be shown at the meeting regarding the time-averaged kinematic vertical wind over Indonesia.

REFERENCES


